

Leçon1 : Calcul numérique partie4

Correction de la serie :4 d'exercices: Les Puissances

Exercice1 : Simplifiez les expressions suivantes ...

$$A = (2^3 \times 2^{-4})^2 \times (3^3)^2 \times 3^{-5} \quad B = 2^3 \times 2^4 \times 2^{-5} \quad C = (2^3 \times 3^2)^2 \quad D = \left(\frac{2}{3}\right)^2 \times 3^3$$

$$E = \left(-\frac{1}{3}\right)^2 \times 5^{-2} \times \left(\frac{3}{5}\right)^3 \quad F = \left(\frac{2}{7}\right)^4 \times \left(\frac{7}{4}\right)^2 \times \left(\frac{-49}{2}\right)^3 \quad G = \left(\frac{2}{3}\right)^{-2} \times \left(\frac{3}{4}\right)^4 \times \left(\frac{27}{4}\right)^{-1}$$

Correction : $A = (2^3 \times 2^{-4})^2 \times (3^3)^2 \times 3^{-5} = (2^{3-4})^2 \times 3^{3 \times 2} \times 3^{-5} = (2^{-1})^2 \times 3^6 \times 3^{-5}$

$$A = 2^{-2} \times 3^{6-5} = 2^{-2} \times 3^1 = \frac{1}{2^2} \times 3 = \frac{3}{4} \quad B = 2^3 \times 2^4 \times 2^{-5} = 2^{3+4-5} = 2^2 = 4$$

$$C = (2^3 \times 3^2)^2 = (2^{3+2})^2 = (2^5)^2 = 2^{5 \times 2} = 2^{10} \quad D = \left(\frac{2}{3}\right)^2 \times 3^3 = \frac{2^2}{3^2} \times 3^3 = 2^2 \times 3^{3-2} = 2^{2+3-2} = 2^3 = 8$$

$$E = \left(-\frac{1}{3}\right)^2 \times 5^{-2} \times \left(\frac{3}{5}\right)^3 = \frac{1}{3^2} \times 5^{-2} \times \frac{3^3}{5^3} = 3^{-2} \times 5^{-2} \times 3^3 \times 5^{-3} = 3^{-2+3} \times 5^{-2+(-3)} = 3^1 \times 5^{-5} = 3 \times 5^{-5} = \frac{3}{5^5} = \frac{3}{3125}$$

$$F = \left(\frac{2}{7}\right)^4 \times \left(\frac{7}{4}\right)^2 \times \left(\frac{-49}{2}\right)^3 = \frac{2^4}{7^4} \times \frac{7^2}{4^2} \times \left(-\frac{7^2}{2}\right)^3 = -\frac{2^4 \times 7^2 \times (7^2)^3}{7^4 \times 4^2 \times 2^3} = -2^4 \times 7^2 \times 7^6 \times 7^{-4} \times 4^{-2} \times 2^{-3}$$

$$F = -2^4 \times 7^2 \times 7^6 \times 7^{-4} \times (2^2)^{-2} \times 2^{-3} = -2^{4-4-3} \times 7^{2+6-4} = -2^{-3} \times 7^4 = -\frac{1}{2^3} \times 7^4 = -\frac{2401}{8}$$

$$G = \left(\frac{2}{3}\right)^{-2} \times \left(\frac{3}{4}\right)^4 \times \left(\frac{27}{4}\right)^{-1} = \left(\frac{3}{2}\right)^2 \times \left(\frac{4}{3}\right)^4 \times \left(\frac{4}{27}\right)^1 = \frac{3^2}{2^2} \times \frac{4^4}{3^4} \times \frac{4}{27} = 3^2 \times 2^{-2} \times (2^2)^2 \times 3^{-4} \times 2^2 \times 3^{-3}$$

$$G = 3^2 \times 2^{-2} \times 2^4 \times 3^{-4} \times 2^2 \times 3^{-3} = 3^{2-4-3} \times 2^{-2+4+2} = 3^{-5} \times 2^4 = \frac{2^4}{3^5} = \frac{16}{243}$$

Exercice2 : simplifier et écrire sous forme d'une puissance

$$A = 2^3 \times (2^2)^4 \times (2^{-5})^3 \quad B = (-3)^1 \times (-3)^5 \times (3)^2 \times (-3)^{-10} \quad C = \frac{3^{-5} \times 4^{-2}}{12^3} \times \frac{9}{2^2} \quad D = \frac{(-2)^3 \times (4^2)^{-1} \times 8}{1024 \times (-16)^{-4}}$$

$$E = \frac{10^{-8} \times 10^9 \times 10^7 \times 10^{-4}}{10^{-2} \times 10^3 \times 10^5} \quad F = \frac{10^{-4} \times (10^3)^2}{10^3}$$

Correction : $A = 2^3 \times (2^2)^4 \times (2^{-5})^3 = 2^3 \times 2^{2 \times 4} \times 2^{-5 \times 3} = 2^{3+8-15} = 2^{-4} \quad A = \frac{1}{2^4} = \frac{1}{16}$

$$B = (-3)^1 \times (-3)^5 \times (3)^2 \times (-3)^{-10} = -(3)^1 \times -(3)^5 \times (3)^2 \times (3)^{-10}$$

$$B = 3^1 \times 3^5 \times 3^2 \times 3^{-10} = 3^{1+5+2-10} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$C = \frac{3^{-5} \times 4^{-2}}{12^3} \times \frac{9}{2^2} = \frac{3^{-5} \times (2^2)^{-2}}{(3 \times 2^2)^3} \times \frac{3^2}{2^2} = \frac{3^{-5} \times (2)^{-4} \times 3^2}{(3)^3 \times 2^6 \times 2^2}$$

$$C = \frac{3^{-5} \times (2)^{-4} \times 3^2}{(3)^3 \times 2^6 \times 2^2} = 3^{-5} \times 2^{-4} \times 3^2 \times (3)^{-3} \times 2^{-6} \times 2^{-2} = 3^{-5-3+2} \times 2^{-4-6-2}$$

$$C = 3^{-6} \times 2^{-12}$$

$$D = \frac{(-2)^3 \times (4^2)^{-1} \times 8}{1024 \times (-16)^{-4}} = \frac{-2^3 \times 4^{2 \times (-1)} \times 2^3}{1024 \times (-2^3)^{-4}} = \frac{-2^3 \times (2^2)^{-2} \times 2^3}{2^{10} \times (-2^3)^{-4}}$$

$$D = -2^3 \times (2^2)^{-2} \times 2^3 \times 2^{-10} \times (-2)^{3 \times 4} = -2^{3-4+3-10+12} = -2^4 = -16$$

$$E = \frac{10^{-8} \times 10^9 \times 10^7 \times 10^{-4}}{10^{-2} \times 10^3 \times 10^5} = 10^{-8} \times 10^9 \times 10^7 \times 10^{-4} \times 10^2 \times 10^{-3} \times 10^{-5}$$

$$E = 10^{-8+9+7-4+2-3-5} = 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01 \quad F = \frac{10^{-4} \times (10^3)^2}{10^3} = 10^{-1} = \frac{1}{10}$$

Exercice3 : Calculer et donner la réponse sans puissances négatives :

$$* 4^3 \times 4^{-5} = \quad * \frac{3^{-5}}{3^{-3}} = \quad * (2^3)^{-2} = \quad * 2^{-3} \times 5^{-3} = \quad * \frac{3^{-2}}{5^{-2}} = \quad * \frac{10^{-3}}{10^2} =$$

Correction : $1/4^2$; $1/3^2$; $1/2^6$; $1/10^3$; $(5/3)^2$; $1/10^6$.

Exercice4 : Ecrire plus simplement : $A = (-2x)^2$ $B = (-2x)^3$ $C = 3x^2y^3 - y(xy)^2$ $D = x^{-1} \times 5x^3$

Correction : $A = (-2x)^2 = 4x^2$ $B = (-2x)^3 = -8x^3$ ~~$C = 3x^2y^3 - y(xy)^2$~~ $= x^2y^3(3 - 1) = 2x^2y^3$

$$D = x^{-1} \times 5x^3 = 5x^2$$

Exercice5 : Ecrire les nombres suivants sous la forme $2^{-j} \times 3^{-k} \times 5^{-l}$

$$150 ; 36 ; \frac{150}{36} ; (150)^2 \times 36 ; \frac{(150)^3}{36} ; \frac{2}{150^2} \left(\frac{6}{5}\right)^2$$

Correction : $150 = 2 \times 3 \times 5^2$ $36 = 6^2 = 2^2 \times 3^2$ $150/36 = 2^{-1} \times 3^{-1} \times 5^2$

$$150^2 \times 36 = (2 \times 3 \times 5^2)^2 \times 2^2 \times 3^2 = 2^4 \times 3^4 \times 5^4$$

$$(150^3)/36 = (2 \times 3 \times 5^2)^3 / (2^2 \times 3^2) = 2 \times 3 \times 5^6$$

$$\frac{2}{150^2} \left(\frac{6}{5}\right)^2 = (2^3 \times 3^2) / ((2 \times 3 \times 5^2)^2 \times 5^2) = 2 \times 5^6$$

Exercice6 : Compléter

$10^3 =$	$10^{-1} =$	$10^{-3} =$
$100 =$	$0,001 =$	$4,3 = 43 \times$
$2,34 = 234 \times$	$0,149 = 149 \times$	$15000 = 15 \times$
$7040 = 704 \times$	$3 \times 10^{-4} =$	$1,4 \times 10^2 =$
$0,012 \times 10^2 =$	$546,3 \times 10^{-2} =$	$2,35 \times 10^{11} = \quad \times 10^9$

Correction :

$10^3 = 1000$	$10^{-1} = 1/10 = 0,1$	$10^{-3} = 0,001$
$100 = 10^2$	$0,001 = 10^{-3}$	$4,3 = 43 \times 10^{-1}$
$2,34 = 234 \times 10^{-2}$	$0,149 = 149 \times 10^{-3}$	$15000 = 15 \times 10^3$
$7040 = 704 \times 10$	$3 \times 10^{-4} = 0,0003$	$1,4 \times 10^2 = 140$
$0,012 \times 10^2 = 1,2$	$546,3 \times 10^{-2} = 5,463$	$2,35 \times 10^{11} = 235 \times 10^9$

Exercice7 : Ecrire les résultats suivants sous forme de multiplication de puissances de 2, 3 et 5 :

a. $\frac{2^2 \times 3^{-4} \times 5}{2 \times 3^2 \times 5^{-3}}$ b. $\frac{6^3 \times 25}{40^2}$

Correction : a. $\frac{2^2 \times 3^{-4} \times 5}{2 \times 3^2 \times 5^{-3}} = 2 \times 3^{-6} \times 5^4$ b. $\frac{6^3 \times 25}{40^2} = \frac{2^3 3^3 5^2}{2^6 5^2} = 2^{-3} 3^3$

Exercice8 : Simplifier les expressions suivantes en montrant les étapes de simplification :

$$A = \frac{10^9 \times 6^3}{25^4 \times 3 \times 2^{11}}, \quad B = \frac{1}{10^{118}} - \frac{1}{10^{119}}, \quad C = 5^{108} \times 2^{106} \times 11 \times \frac{1}{10^{107}}$$

Correction : $A = 90$, $B = 9 \times 10^{-119}$, $C = \frac{55}{2}$ en effet :

$$A = \frac{10^9 \times 6^3}{25^4 \times 3 \times 2^{11}} = \frac{(5 \times 2)^9 \times (3 \times 2)^3}{(5^2)^4 \times 3 \times 2^{11}} = \frac{5^9 \times 2^9 \times 3^3 \times 2^3}{5^8 \times 3^1 \times 2^{11}} = \frac{5^9 \times 2^{9+3} \times 3^3}{5^8 \times 3^1 \times 2^{11}} = 5^9 \times 2^{12} \times 3^3 \times 5^{-8} \times 3^{-1} \times 2^{-11}$$

$$A = 5^{9-8} \times 2^{12-11} \times 3^{3-1} = 5 \times 2 \times 9 = 10 \times 9 = 90$$

$$B = \frac{1}{10^{118}} - \frac{1}{10^{119}} = \frac{10}{10^{118} \times 10} - \frac{1}{10^{119}} = \frac{10^1}{10^{119}} - \frac{1}{10^{119}} = \frac{10-1}{10^{119}} = \frac{9}{10^{119}} = 9 \times 10^{-119}$$

$$C = 5^{108} \times 2^{106} \times 11 \times \frac{1}{10^{107}} = 5^{106} \times 5^2 \times 2^{106} \times 11 \times \frac{1}{10^{107}} = (5 \times 2)^{106} \times 5^2 \times 11 \times \frac{1}{10^{107}} = 10^{106} \times 25 \times 11 \times 10^{-107}$$

$$C = 10^{106-107} \times 25 \times 11 = 10^{-1} \times 25 \times 11 = \frac{5 \times 5 \times 11}{10} = \frac{5 \times 5 \times 11}{2 \times 5} = \frac{5 \times 11}{2} = \frac{55}{2}$$

Exercice9 : a , b et c sont des nombres non nuls. Ecrire les nombres suivants sous la forme

$$a^p \times b^q \times c^r : A = \frac{c}{\left(\frac{a}{b}\right)^2} \quad B = a^5 (bc)^2 \times \frac{1}{(a^3 b)^2} \quad C = \frac{ab^2}{ca^{-2}} \quad D = (a^3 b^{-5})^2$$

Correction : $A = \frac{c}{\left(\frac{a}{b}\right)^2} = a^{-2} \times b^2 \times c$; $B = a^5 (bc)^2 \times \frac{1}{(a^3 b)^2} = a^{-1} \times c^2$

$$C = \frac{ab^2}{ca^{-2}} = a^3 \times b^2 \times c^{-1} \quad ; \quad D = (a^3 b^{-5})^2 = a^6 \times b^{-10}.$$

Exemple10 : Écriture scientifique d'un nombre décimal

La notation scientifique d'un nombre décimal est de la forme $a \times 10^p$ où a est un nombre décimal ($1 \leq a < 10$) et p un nombre entier relatif.

Mettre en notation scientifique :

$$24,5 = \quad 4500 = \quad 0,0078 = \quad -658 = \quad 0,000085 = \quad -7005000 =$$

Correction : $24,5 = 2,45 \cdot 10$ $-658 = -6,58 \cdot 10^2$ $4500 = 4,5 \cdot 10^3$
 $0,000085 = 8,5 \cdot 10^{-5}$ $0,0078 = 7,8 \cdot 10^{-3}$ $-7005000 = -7,005 \cdot 10^6$

Exercice11 : Les constantes universelles : les formules de physique comportent souvent des nombres très particuliers que l'on appelle constantes universelles (par exemple la célérité de la lumière : $c = 299\,792\,458 \text{ m s}^{-1}$).

Ecrire les constantes universelles suivantes en notations scientifiques :

$$F = 96484,56 \quad u = 166,0565 \times 10^{-29} \quad h = 0,6626176 \times 10^{-33} \quad c = 299792458$$

$$g = 980,665 \times 10^{-2}; \quad N_A = 6\,022,045 \times 10^{20}; \quad m_e = 910,9534 \times 10^{-33}; \quad e = 1602,1892 \times 10^{-22}$$

Correction : $F = 96484,56 = 9,648456 \times 10^4$ $u = 166,0565 \times 10^{-29} = 1,660565 \times 10^2 \times 10^{-29} = 1,660565 \times 10^{-27}$

$$h = 0,6626176 \times 10^{-33} = 6,626176 \times 10^{-1} \times 10^{-33} = 6,626176 \times 10^{-34} \quad c = 2,99792458 \times 10^8$$

$$g = 980,665 \times 10^{-2} = 9,80665 \times 10^2 \times 10^{-2} = 9,80665 \times 10^0$$

$$N_A = 6\,022,045 \times 10^{20} = 6,022045 \times 10^3 \times 10^{20} = 6,022045 \times 10^{23}$$

$$m_e = 910,9534 \times 10^{-33} = 9,109534 \times 10^2 \times 10^{-33} = 9,109534 \times 10^{-31}$$

$$e = 1602,1892 \times 10^{-22} = 1,6021892 \times 10^3 \times 10^{-22} = 1,6021892 \times 10^{-19}$$